Commentary

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# Relevant and Robust: A Response to Marcus and Davis (2013)

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Computational models in psychology are precise, fully explicit scientific hypotheses. Probabilistic models in particular formalize hypotheses about the beliefs of agents—their knowledge and assumptions about the world—using the structured collection of probabilities referred to as priors and likelihoods. The probability calculus then describes inferences that can be drawn by combining these beliefs with new evidence, without the need to commit to a process-level explanation of how these inferences are performed (Marr, 1982). Over the past 15 years, probabilistic modeling of human cognition has yielded quantitative theories of a wide variety of phenomena (Tenenbaum, Kemp, Griffiths, & Goodman, 2011).

Marcus and Davis (2013) critiqued several examples of this work on the basis of the existence of alternative models and potentially inconsistent data, using these critiques to question the basic validity of the probabilistic-models approach. Contra the broad rhetoric of their article, however, the points made by Marcus and Davis—although useful to consider—do not indicate systematic problems with the probabilistic-modeling enterprise.

Several of Marcus and Davis's objections stem from a fundamental confusion about the status of optimality in probabilistic modeling, which has been discussed in responses to other critiques (see Frank, 2013; Griffiths, Chater, Norris, & Pouget, 2012). Briefly, *an* optimal analysis is not *the* optimal analysis for a task or domain. Different probabilistic models instantiate different psychological hypotheses. Optimality provides a bridging assumption between these hypotheses and human behavior—one that can be reexamined or overturned as the data warrant.

## Model Selection

Marcus and Davis argued that individual probabilistic models require a host of potentially problematic modeling choices. Indeed, probabilistic models are created via a series of choices concerning, among other things, priors, likelihoods, and response functions. Each of these choices embodies a proposal about cognition, and these proposals will often be wrong. Identifying model assumptions that result in a mismatch to empirical data allows these assumptions to be replaced or refined.

Systematic iteration to achieve a better model is part of the normal progress of science. But if choices are made post hoc, a model can be *overfit* to the particulars of the empirical data. Marcus and Davis suggested that certain of our models suffer from this problem. For instance, they showed that data on pragmatic inference (Frank & Goodman, 2012) are inconsistent with an alternative variant of the proposed model that uses a hard-max rather than a soft-max function, and they asked whether our choice of a soft-max rule was dependent on the data.

The soft-max rule is foundational in economics, decision theory, and cognitive psychology (Luce, 1959, 1977), and we first selected it for this problem on the basis of a completely independent set of experiments (Frank, Goodman, Lai, & Tenenbaum, 2009). It is therefore hard to see how a claim of overfitting is warranted. Modelers must balance unification with exploration of model assumptions across tasks, but this issue is a general one for all computational work and does not constitute a systematic problem with the probabilistic approach.

## Task Selection

Marcus and Davis suggested that probabilistic modelers report results on only the narrow range of tasks on which

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their models succeed. But their critique focused on a few high-profile, short reports that represented our first attempts to engage with important domains of cognition. Such reports necessarily have less in-depth engagement with empirical data than more extensive and mature work, though they also exemplify the applicability of probabilistic modeling to domains previously viewed as too complex for quantitative approaches.

There is broader empirical adequacy to probabilistic models of cognition than Marcus and Davis implied. If they had surveyed the literature, they would have found substantial additional evidence for the models they reviewed—and more has accrued since their critique. For example, Marcus and Davis critiqued Griffiths and Tenenbaum's (2006) analysis of everyday predictions for failing to provide independent assessments of the contributions of priors and likelihoods, precisely what was done in several later and much longer articles (Griffiths & Tenenbaum, 2011; Lewandowsky, Griffiths, & Kalish, 2009). They similarly critiqued the particular tasks selected by Battaglia, Hamrick, and Tenenbaum (2013) without discussing the growing literature—some published prior to Marcus and Davis's response and some after—testing similar "noisy Newtonian" models on other phenomena (Gerstenberg, Goodman, Lagnado, & Tenenbaum, 2012, 2014; Sanborn, Mansinghka, & Griffiths, 2013; Téglás et al., 2011). In newer work, Smith, Battaglia, and Vul (2013) even directly addressed exactly the challenge Marcus and Davis posed regarding classic findings of errors in physical intuitions. In other domains, such as concept learning and inductive inference, where there is an extensive experimental tradition, probabilistic models have engaged with diverse empirical data collected by multiple labs over many years (e.g., Goodman, Tenenbaum, Feldman, & Griffiths, 2008; Kemp & Tenenbaum, 2009).

Marcus and Davis also insinuated empirical problems that they did not test. For instance, in criticizing Frank and Goodman's (2012) choice of dependent measure, they posited that a forced-choice task would yield a qualitatively different pattern (discrete rather than graded responding). In fact, a forced-choice version of the task produces graded patterns of responding across a wide variety of conditions (Stiller, Goodman, & Frank, 2011, 2014; Vogel, Emilsson, Frank, Jurafsky, & Potts, 2014).

## Conclusions

We agree with Marcus and Davis that there are real and important challenges for probabilistic models of cognition, as there will be for any approach to modeling a system as complex as the human mind. To us, the most pressing challenges include understanding the relationship of such models to lower levels of psychological analysis and neural implementation, integrating additional formal tools, clarifying the philosophical status of the models, extending them to new domains of cognition, and, yes, engaging with additional empirical data in the current domains while unifying specific model choices into broader principles. As Marcus and Davis stated, "ultimately, the Bayesian approach should be seen as a useful tool" (p. 2358)—and we believe that it has already proven its robustness and relevance by allowing us to form and test quantitatively accurate psychological hypotheses.

### Author Contributions

All authors contributed to writing this manuscript.

### Declaration of Conflicting Interests

The authors declared that they had no conflicts of interest with respect to their authorship or the publication of this article.

### References

Battaglia, P. W., Hamrick, J. B., & Tenenbaum, J. B. (2013). Simulation as an engine of physical scene understanding. *Proceedings of the National Academy of Sciences, USA*, *110*, 18327–18332.

Frank, M., Goodman, N., Lai, P., & Tenenbaum, J. (2009). Informative communication in word production and word learning. In N. Taatgen & H. van Rijn (Eds.), *Proceedings of the 31st Annual Conference of the Cognitive Science Society* (pp. 1228–1233). Austin, TX: Cognitive Science Society.

Frank, M. C. (2013). Throwing out the Bayesian baby with the optimal bathwater: Response to Endress (2013). *Cognition*, *128*, 417–423.

Frank, M. C., & Goodman, N. D. (2012). Predicting pragmatic reasoning in language games. *Science*, *336*, 998.

Gerstenberg, T., Goodman, N., Lagnado, D., & Tenenbaum, J. (2012). Noisy Newtons: Unifying process and dependency accounts of causal attribution. In N. Miyake, D. Peebles, & R. P. Cooper (Eds.), *Proceedings of the 34th Annual Conference of the Cognitive Science Society* (pp. 378–383). Austin, TX: Cognitive Science Society.

Gerstenberg, T., Goodman, N., Lagnado, D., & Tenenbaum, J. (2014). From counterfactual simulation to causal judgment. In P. Bello, M. Guarini, M. McShane, & B. Scassellati (Eds.), *Proceedings of the 36th Annual Meeting of the Cognitive Science Society* (pp. 523–528). Austin, TX: Cognitive Science Society.

Goodman, N. D., Tenenbaum, J. B., Feldman, J., & Griffiths, T. L. (2008). A rational analysis of rule-based concept learning. *Cognitive Science*, *32*, 108–154.

Griffiths, T. L., Chater, N., Norris, D., & Pouget, A. (2012). How the Bayesians got their beliefs (and what those beliefs actually are): Comment on Bowers and Davis (2012). *Psychological Bulletin*, *138*, 415–422.

Griffiths, T. L., & Tenenbaum, J. B. (2006). Optimal predictions in everyday cognition. *Psychological Science*, *17*, 767–773.

Griffiths, T. L., & Tenenbaum, J. B. (2011). Predicting the future as Bayesian inference: People combine prior knowledge with observations when estimating duration and extent. *Journal of Experimental Psychology: General*, *140*, 725–743.

Kemp, C., & Tenenbaum, J. B. (2009). Structured statistical models of inductive reasoning. *Psychological Review*, *116*, 20–58.

Lewandowsky, S., Griffiths, T. L., & Kalish, M. L. (2009). The wisdom of individuals: Exploring people's knowledge about everyday events using iterated learning. *Cognitive Science*, *33*, 969–998.

Luce, R. D. (1959). *Individual choice behavior: A theoretical analysis*. New York, NY: Wiley.

Luce, R. D. (1977). The choice axiom after twenty years. *Journal of Mathematical Psychology*, *15*, 215–233.

Marcus, G. F., & Davis, E. (2013). How robust are probabilistic models of higher-level cognition? *Psychological Science*, *24*, 2351–2360.

Marr, D. (1982). *Vision*. Cambridge, MA: MIT Press.

Sanborn, A. N., Mansinghka, V. K., & Griffiths, T. L. (2013). Reconciling intuitive physics and Newtonian mechanics for colliding objects. *Psychological Review*, *120*, 411–437.

Smith, K., Battaglia, P., & Vul, E. (2013). Consistent physics underlying ballistic motion prediction. In M. Knauff, M. Pauen, N. Sebanz, & I. Wachsmuth (Eds.), *Proceedings of the 35th Annual Conference of the Cognitive Science Society* (pp. 3426–3431). Austin, TX: Cognitive Science Society.

Stiller, A., Goodman, N. D., & Frank, M. C. (2011). Ad-hoc scalar implicature in adults and children. In L. Carlson, C. Hoelscher, & T. F. Shipley (Eds.), *Proceedings of the 33rd Annual Conference of the Cognitive Science Society* (pp. 2134–2139). Austin, TX: Cognitive Science Society.

Stiller, A. J., Goodman, N. D., & Frank, M. C. (2014). Ad-hoc implicature in preschool children. *Language Learning and Development*. Advance online publication. doi:10.1080/15475441.2014.927328

Téglás, E., Vul, E., Girotto, V., Gonzalez, M., Tenenbaum, J. B., & Bonatti, L. L. (2011). Pure reasoning in 12-month-old infants as probabilistic inference. *Science*, *332*, 1054–1059.

Tenenbaum, J. B., Kemp, C., Griffiths, T. L., & Goodman, N. D. (2011). How to grow a mind: Statistics, structure, and abstraction. *Science*, *331*, 1279–1285.

Vogel, A., Emilsson, A. G., Frank, M. C., Jurafsky, D., & Potts, C. (2014). Learning to reason pragmatically with cognitive limitations. In P. Bello, M. Guarini, M. McShane, & B. Scassellati (Eds.), *Proceedings of the 36th Annual Meeting of the Cognitive Science Society* (pp. 3055–3060). Austin, TX: Cognitive Science Society.